Dispersion models for brine discharges from desalination plants of Oman

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ABSTRACT

Desalination of seawater leaves brine waste containing high salt concentration to be disposed of into the environment. For coastal desalination plants, there is no better alternative than to continuously discharge brine into the sea via an outfall. Subsequent motions, mixing and spreading of the brine are important factors in assessing the impact of disposal operations. Modeling studies of the effect of seabed depth profiles upon dispersing brine discharges into the sea are conducted using a two-dimensional advection-diffusion equation. For illustrations, simple depth profiles of a uniformly sloping beach, a vertical beach with a constant water depth and a step beach with a depth change in its profile are used. Solutions are presented graphically by plotting contours of concentration, demonstrating that if the outfall is located close to the beach, the brine plumes spread towards the coastal area. However, it is found that without building a longer outfall, the coastline brine concentration can be drastically reduced by creating a step discontinuity on the seabed depth profile.

Keywords: Advection-diffusion equation; brine disposal; Oman; sea outfall; seawater desalination.

INTRODUCTION

In the arid climate of Oman, as in the rest of the Gulf countries, it is no longer possible to rely solely on underground water resources to satisfy the continuously growing demands for water (MHEW 2002). A sharp increase in water consumption is expected in parallel with the rapid development of industry, cities and agriculture along the coastal areas. As a result, not only is underground water over-exploited and over-pumped, its quality is also deteriorating as a result of seawater intrusion. It is therefore necessary for large coastal cities to turn to desalination of seawater to provide reliable supplies of water.

Desalination separates seawater into freshwater and brine, hot wastewater
containing high salt concentration. The largest seawater desalination plant in Oman is located in Ghubrah, and its daily production capacity is 42 million gallons of water, but it is continuously discharging its brine waste directly into the Gulf of Oman in excess of 28 million gallons per day. Similarly, many other smaller coastal desalination plants in Oman also dispose of their waste brine into the sea (Fig. 1).

It is commonly assumed that brine discharge will ultimately be diluted into the sea. However, due to the relatively shallow water depth, the brine plumes are observed to be drifting along and slowly spreading towards the coast. This will therefore cause an increase in salinity in the coastal waters, and for Oman, will enhance the severe problems of seawater intrusion into coastal underground aquifers (Purnama et al. 2003). The other possible environmental impact commonly associated with brine discharges into the sea is that concentrated brines may sink to the ocean floor and threaten the benthic environment (Hopner & Windelberg 1996, Mabrook 1994), which in turn, may affect the productivity of fisheries resources, and may also destroy coral reefs.

A coastal area is the dynamic region where land and sea meet. In some places, it is a mountainous coast with rocky sea cliffs, where the land thruts out into the sea and the coastal waters get deep very rapidly. Elsewhere, it is a sandy beach, where the sea gently extends into the land. Coastal areas and beaches are important to Oman for fisheries, local recreation and tourism, as well as providing breeding grounds for loggerhead turtles, and feeding grounds for migratory birds. Major coral growth takes place along the rocky coast of the Gulf of Oman and the offshore islands of Al Halaaniyat in the Arabian Sea.
Oman has a coastline stretching for more than 1700 km from the Gulf of Oman in the north to the Arabian Sea in the south (Fig. 1). Beginning at the northern end of Oman, the Musandam coast is characterized by precipitous slopes that continue below water to depths exceeding 40 m. The northern coastline of Oman are predominantly sandy beaches which follow south from Ghubrah to the eastern tip of the Arabian peninsula with sandy beaches and rocky coastal cliffs. The coastline exhibits a steep bathymetry to depths greater than 2000 m within 8 km offshore. The southern part of the Omani coastal waters face the Arabian Sea and extend to the Yemeni border. The southern coast is mostly sandy beaches, but further south towards the border of Yemen, the coast is characterized by low metamorphic rocks and cliffs with steep slopes descending into the sea.

If continuous brine discharges into the sea cannot be avoided, then it should be done as optimally as possible to ensure that the environmental impact is minimized. Therefore, it is important to know how the brines are mixed and dissolved into the sea. So far, little information is available on the environmental impact assessments, and the only field assessment studies have recently been reported by Talavera & Ruiz (2001) for the desalination plant in the Canary Islands. Unfortunately, there are also not many mathematical models available for evaluating the environmental impact of brine discharges into the sea. The CORMIX computer code adapted for modeling brine discharges was first reported by Del Bene et al. (1994). However, no details are given on the model assumption and equations. The most updated version of CORMIX-GI systems was recently reported by Doneker & Jirka (2001). Although CORMIX may simulate plume behaviour in the immediate vicinity of the outfall, environmental impacts will typically be observed in areas that are a relatively large distance away from the outfall.

One factor affecting the dispersion of brine discharges is the seabed depth profile. In deeper water, the mixing is stronger as the current tends to be faster, and there is a greater depth over which to dilute brine. Modeling studies of the effect of water depth variations upon dispersing brine discharges into the sea are investigated using a two-dimensional advection-diffusion equation. We first consider simple profiles of a uniformly sloping beach (Kay 1987), and a vertical beach with a constant water depth to model a mountainous coast with rocky sea cliffs. Then a step beach with a depth change in its profile is introduced. It is the standard practice to build a longer sea outfall in order to minimize the coastline brine concentration. However, for a desalination plant, this solution will complicate the plant design requirement since the seawater intake should be kept as far away as possible from the brine waste outfall.
MODEL ASSUMPTIONS

Since we are only concerned with the effect of the seabed depth profile on brine discharge dispersion, other factors such as tidal motions, temperature and density are ignored. The coastline is assumed to be straight, and brine is continuously being discharged with a constant rate $Q$ at $(0, \alpha h_0)$, where $h_0$ is an arbitrary reference water depth. As the discharges are made through diffusers, it is also assumed that the brine is vertically well-mixed over the water depth, $h$ (see Talavera & Ruiz 2001).

Waves at sea change their headings, always turning toward shallower coastal waters where they slow down. As waves approach the coast at an angle, the shallow seabed refracts and bends the wave fronts. The refracted waves generate the net coast parallel-component (longshore) currents. The current is assumed to be steady with speed $U$ and remain in the $x$-direction at all times. The dispersion mechanisms are represented by eddy diffusivities, and diffusion in the $x$-direction is neglected, as the brine plumes in steady currents become very elongated in the $x$-direction. For illustration, the variation in the $y$-direction of $U$ is assumed proportional to $h^{1/2}$ and coefficient of diffusivity $D$ to $h^{3/2}$.

UNIFORMLY SLOPING BEACH MODEL

Seawater desalination plants are built predominantly on sloping sandy beaches, where $h$ increases linearly with $y$, i.e. $h = my$. The beach is at $y = 0$ with slope $m$ (Fig. 2a). The advection-diffusion equation for the concentration $c$ is

$$hU \frac{\partial c}{\partial x} - \frac{\partial}{\partial y} \left( hD \frac{\partial c}{\partial y} \right) = Q \delta(x) \delta(y - \alpha h_0),$$

(1)

![Fig.2. Seabed depth profiles. (a) Uniformly sloping beach, and (b) Vertical beach.](image-url)
with the boundary condition \( hD \frac{\partial c}{\partial y} = 0 \) at \( y = 0 \) and the brine is assumed to be completely dissolved into the sea. \( \delta \) is the Dirac delta function.

For a graphical representation of the results, we define dimensionless quantities \( y = y^* h_0, \ x = x^* U_0 h_0^2 / D_0 \) and \( c = c^* Q / h_0^3 U_0 \). If we take \( U = U_0 y^*^{1/2} \) and \( D = D_0 y^*^{3/2} \), the solution of (1), in its dimensionless form, is

\[
c^* = \frac{1}{m x^*} \left( \frac{1}{\alpha y^*} \right)^{3/4} \exp \left( -\frac{y^* + \alpha}{x^*} \right) I_{3/2} \left( \frac{2\sqrt{\alpha y^*}}{x^*} \right),
\]

(2)

where \( I_{3/2} \) is a modified Bessel function. Derivations of (2) can be found in Kay (1987).

Contours of concentration (2) were plotted in Fig. 3(a) for \( m = 0.15 \) and \( \alpha = 3 \). As the water depth is gradually decreasing towards the beach, the plumes are observed to be spreading towards the beach. Note that the actual brine plumes are elongated in the \( x \)-direction, and if we take \( D_0 \approx 0.01 h_0 U_0 \), then the elongation factor is of the order of 100. That is, if the length scale in the \( y \)-direction is of the order 10 m, then the length scale in the \( x \)-direction is of the order 1 km.

The most sensitive location for assessing the impact of brine discharges into the sea would be at the beach. By letting \( y^* \rightarrow 0 \) and replacing \( I_{3/2} \) in (2) by its limiting form, we obtain the concentration at the beach \( c^*(0) = \frac{4}{3\sqrt{\pi} m x^*^{5/2}} \exp \left( -\frac{\alpha}{x^*} \right) \). As shown in Fig. 3(b), it has a maximum value \( c^*(0)_{\text{max}} \approx 0.61 / m \alpha^{5/2} \), which occurs at \( x^* = 2\alpha / 5 \).

![Fig.3. Uniformly sloping beach with \( m = 0.15 \) and \( \alpha = 3 \). (a) Contours of concentration, and (b) Concentration at the beach for the beach slope \( m = 0.3 \) (- - -) and \( m = 0.15 \) (-----).](image-url)
However, the maximum concentration at the beach can be reduced by increasing the value of $\alpha$, i.e. by building a longer sea outfall. Physically what happens is that relatively low currents delay the transport of brine towards the beach, while diffusion away from the beach continues to reduce the brine concentration.

The cost of building a long sea outfall is very significant, and for the existing seawater desalination plant, extending the outfall is impractical, since it would require some changes to be made to the plant design. In particular, the seawater intake location must be kept as far away as possible from the brine outfall. As reported by Talavera & Ruiz (2001), the desalination plant on the Canary Islands has a long brine outfall 300 m away from the beach and 600 m from the seawater intake pipes. If we assume that the beach is uniformly sloping with slope of $m = 0.15$, the bathing area criteria of maximum concentration at the beach equal to $10^{-3}$ could be achieved for $\alpha = 25$. Thus, we obtain $h_0 = 12$ m. However, the price paid for having such a long brine outfall is that the intake seawater to the plant has unusually high salinity. As shown by Talavera & Ruiz (2001, Figs. 5 and 6), instead of spreading towards the beach, the brine plumes spread past the intake pipes. Finally, for the case of a long sea outfall, the uniformly sloping beach assumption is no longer held, as the seabed depth profile is typically depicted in the oceanography textbooks as a gentle slope coming in contact with a steeper one (see Fig. 3 of Talavera & Ruiz 2001).

**VERTICAL BEACH MODEL**

Let’s consider a simple seabed depth profile of a vertical beach of a constant water depth, $h = h_0$ (Fig. 2b). The beach at $y = 0$ is assumed to be a continuation of the rock sea cliff, which is an impermeable vertical wall, and the boundary condition is $D \frac{\partial c}{\partial y} = 0$ at $y = 0$. In this highly simplified depth profile, both $U_0 \propto h_0^{1/2}$ and $D_0 \propto h_0^{3/2}$ are constants.

Using the method of reflection, if the brine is discharged with a rate $Q$ at $(0, \alpha h_0)$, then the boundary condition is automatically satisfied by the introduction of an imaginary source at $(0, -\alpha h_0)$ discharging with the same rate $Q$. So instead of (1), the advection-diffusion equation is

$$U_0 \frac{\partial c}{\partial x} - D_0 \frac{\partial^2 c}{\partial y^2} = \frac{Q}{h_0} \delta(x)[\delta(y - \alpha h_0) + \delta(y + \alpha h_0)]. \quad (3)$$
The solution, in its dimensionless form, is

\[
c* = \frac{1}{2\sqrt{\pi x^*}} \left[ \exp\left(-\frac{(y^* - \alpha)^2}{4x^*}\right) + \exp\left(-\frac{(y^* + \alpha)^2}{4x^*}\right) \right]. \tag{4}
\]

Contours of concentration (4) were plotted in Fig. 4(a) for \( \alpha = 2 \). Due to the impermeable vertical beach at \( y = 0 \), the brine plumes are observed to be gradually spreading towards the beach.

**Fig. 4.** Vertical beach with a constant water depth. (a) Contours of concentration for \( \alpha = 2 \), and (b) Concentration at the beach for \( \alpha = 4 \) (——) and \( \alpha = 5 \) (— — —).

**Fig. 5.** Contours of concentration on a vertical beach for \( \alpha = 5 \). The full solution is shown by (— — —).
By putting \( y^* = 0 \) in (4), we obtain the concentration at the beach
\[
c^*(0) = \frac{1}{\sqrt{\pi x^*}} \exp\left( -\frac{\alpha^2}{4x^*} \right).
\]
As shown in Fig. 4(b), it has a maximum value of
\[
c^*(0)_{\text{max}} = \sqrt{\frac{2}{\pi e}} / \alpha,
\]
which occurs at a longer distance \( x^* = \alpha^2 / 2 \). Therefore, although the concentration at the beach can be reduced by increasing the value of \( \alpha \), the brine plumes are more elongated in the \( x \)-direction. The plumes must have traveled a very long distance in order to reach the beach. This may also be interpreted as the plumes do not feel the presence of the vertical beach at \( y = 0 \). Thus for \( \alpha \geq 5 \), we can neglect the contribution of the imaginary source at \((0, -\alpha h_0)\) from (4). As illustrated in Fig. 5 for \( \alpha = 5 \), this is a good approximation for \( x^* \leq 3 \), which is equivalent to a downstream distance of 3 km from the outfall. Talavera & Ruiz (2001) collected samples within a radius of 2 km from the brine outfall.

**STEP BEACH MODEL**

Unfortunately, the vertical beach profile is not realistic. We now extend it to account for the presence of a depth change across the line \( y = 0 \) (Fig. 6), where the water depth \( h_0 \) in the near-shore region \(-\ell h_0 < y < 0\) is shallower than \( h_1 \) in the deeper sea region \( y > 0 \). By writing \( c = \begin{cases} c_0(x, y), & -\ell h_0 < y < 0 \\ c_1(x, y), & y > 0 \end{cases} \), the solution of the advection-diffusion equation can be obtained using the method of images (Kay 1987). So if the outfall is located in the shallow region \(-\ell h_0 < y < 0\), then by treating the line \( y = 0 \) as a reflecting boundary, the solution \( c_0 \) is obtained due to a combination of discharges at \((0, -\alpha_1 h_0)\) with a rate \( Q \), and with a rate \( aQ \) from an image source at \((0, \alpha_1 h_0)\). The outfall is now

![Fig.6. Seabed depth profile of a step beach.](image-url)
located at a distance \((\ell - \alpha_1)h_0\) from the beach, and from the previous section, the effect of the impermeable vertical beach at \(y = -\ell h_0\) can be neglected within \(x^* \leq 3\), if we assume that \(\ell \geq 6\) and \(\alpha_1 \leq 1\). Therefore, the advection-diffusion equation for the concentration \(c_0\) is

\[
U_0 \frac{\partial c_0}{\partial x} - D_0 \frac{\partial^2 c_0}{\partial y^2} = \frac{Q}{h_0} \delta(x)[\delta(y + \alpha_1 h_0) + a \ \delta(y - \alpha_1 h_0)].
\]  

\( (5) \)

The solution, in dimensionless form, is

\[
c_{0^*} = \frac{1}{2\sqrt{\pi x^*}} \left[ \exp \left( -\frac{(y^* + \alpha_1)^2}{4x^*} \right) + a \ \exp \left( -\frac{(y^* - \alpha_1)^2}{4x^*} \right) \right].
\]  

\( (6) \)

In the deeper region, the line \(y = 0\) is treated as an absorbing boundary, and the solution \(c_1\) is obtained due to a virtual source at \((0, -\beta h_0)\) discharging with a rate \(bQ\). So the advection-diffusion equation for the concentration \(c_1\) is

\[
U_1 \frac{\partial c_1}{\partial x} - D_1 \frac{\partial^2 c_1}{\partial y^2} = \frac{bQ}{h_1} \delta(x) \ \delta(y + \beta h_0).
\]  

\( (7) \)

If we take \(U_1 = U_0 h^*^{1/2}\) and \(D_1 = D_0 h^*^{3/2}\), where \(h^* = h_1/h_0\) is the water depth ratio, then the solution, in dimensionless form, is

\[
c_{1^*} = \frac{b}{2h^*^2 \sqrt{\pi x^*}} \exp \left( -\frac{(y^* + \beta)^2}{4h^* x^*} \right).
\]  

\( (8) \)

If \(h^* = 1\) (i.e. there is no depth change) then \(a = 0, b = 1\) and \(\beta = \alpha_1\), and solutions (6) and (8) are reduced to that of the vertical beach solution (4).

However, as there can be no sharp discontinuities in either the concentration or its gradient across \(y = 0\), the additional conditions to be satisfied in order to calculate the coefficients \(a, b\) and \(\beta\) are

\[
\lim_{y \to 0^-} c_0 = \lim_{y \to 0^+} c_1 \quad \text{and} \quad \lim_{y \to 0^-} h_0 D_0 \frac{\partial c_0}{\partial y} = \lim_{y \to 0^+} h_1 D_1 \frac{\partial c_1}{\partial y}.
\]  

\( (9) \)

First, \(\beta\) is obtained by equating the leading exponential terms in (6) and (8), i.e. \(\beta = \alpha_1 \sqrt{h^*}\), and then \(a\) and \(b\) by solving the resulting two linear equations: \(a = (1 - h^*)/(1 + h^*)\) and \(b = 2h^*/(1 + h^*)\). Note that, \(a \leq 0\) and \(\beta \geq \alpha_1\) as \(h^* \geq 1\).

Contours of concentrations (6) and (8) were plotted for \(h^* = 2\) in Fig. 7(a),
where \( \ell = 6 \) and \( \alpha_1 = -1 \). As the water depth in the shallow region \( -\ell h_0 < y < 0 \) is half of that in the deeper region \( y > 0 \), only a small part of the plumes spread towards the beach. By substituting \( y = -\ell \) in (6), the concentration at the step beach is less than that of the vertical beach by

\[
al = \frac{(1 - h^*)^2}{(1 + h^*)^2}.
\]

Furthermore, as shown in Fig. 7(b), the concentration at the step beach with \( h^* = 2 \) is also less than that of the uniformly sloping beach with \( m = 0.15 \) and \( \alpha = 6 \).

Similarly, if the outfall is located in the deeper region \( y > 0 \) at a distance \( (\ell + \alpha_1)h_0 \) from the beach, then the line \( y = 0 \) is treated as a reflecting boundary for the concentration \( c_1 \), and as an absorbing boundary for \( c_0 \). The solutions are then obtained in a similar manner, simply by exchanging the subscript 0 to 1, \( \alpha_1 \) to \( -\alpha_1 \) and \( -\beta \) to \( \beta \). Thus, the concentration \( c_1 \) is obtained by solving equation (5), and in dimensionless form,

\[
c_1^* = \frac{1}{2h^*} \sqrt{\frac{\pi}{x^*}} \left[ \exp \left( -\frac{(y^* - \alpha_1)^2}{4h^* x^*} \right) + a \exp \left( -\frac{y^* + \alpha_1)^2}{4h^* x^*} \right) \right].
\]  

By interchanging the subscripts, we introduce the water depth ratio \( h^* \) into the solution. The concentration \( c_0 \) is obtained by solving equation (7), and in dimensionless form,

\[
c_0^* = \frac{b}{2\sqrt{\pi x^*}} \exp \left( -\frac{(y^* - \beta)^2}{4x^*} \right).
\]
Fig. 8. Step beach with $h^* = 2$ and $\ell = 6$. (a) Contours of concentration for $\alpha_1 = 1$, and (b) Concentration at the beach. The uniformly sloping beach with $m = 0.15$ and $\alpha = 8$ is shown by (---).

The matching conditions (9) may now be applied, and it is found that $\beta = \alpha_1 / \sqrt{h^*}$, $a = (h^*^2 - 1)/(1 + h^*^2)$ and $b = 2/(1 + h^*^2)$. Note that, $a \geq 0$ and $\beta \leq \alpha_1$ as $h^* \geq 1$. Contours of concentrations (10) and (11) were plotted for $h^* = 2$ in Fig. 8(a), where $\ell = 6$ and $\alpha_1 = 1$. Instead of spreading towards the beach, the plumes are more elongated in the $x$-direction, and the concentration at the step beach is $c_{00}(-\ell) = \frac{1}{(1+h^*^2)\sqrt{\pi h^*}} \exp\left[-\frac{(\ell + \alpha_1 \sqrt{h^*})^2}{4h^*}\right]$. In comparison with that of the vertical beach, the maximum concentration at the step beach is reduced by a factor $1/(1 + h^*^2)$. Thus, as shown in Fig. 8(b), the concentration at the step beach with $h^* = 2$ is even less than that of the uniformly sloping beach with $m = 0.15$ and $\alpha = 8$.

CONCLUSIONS

The effect of the seabed depth profiles upon mixing and spreading of brine continuously being discharged into the sea are investigated using a two-dimensional advection-diffusion equation. Simple models of depth profile are used, including a uniformly sloping beach, a vertical beach with a constant water depth, and a step beach with a depth change in its profile. By plotting contours of brine concentration, it is found that if the outfall is located near the beach, the brine plumes are spreading towards the beach. However, the concentration at the beach can be drastically reduced by either discharging brine further out to sea, or by creating a step discontinuity on its depth profile. The solutions obtained can easily be modified to any type of pollutant continuously
being discharged into the sea via an outfall, including treated effluents from coastal sewage works, toxic contaminants from industrial installations in coastal areas, and hot water discharged from coastal power stations.

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نماذج متعددة للتخلص من الأملاح

الثابتة عن محطات تحلية مياه البحر في عمان

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خلاصة

تؤدي عملية تحلية مياه البحر إلى الحصول على مياه مركزة الملوحة والتي يتم التخلص منها في البيئة. إن هذه العملية تتم بواسطة محطات التحلية الواقعة على ساحل البحر. ولذا فإن أفضل طريقة للتخلص من تلك المياه المالحة هي صبها في ساحل المحطة بشكل مستمر.

ومن المهم التعرف على عملية اختلاط، وانتشار المياه المالحة لتقييم أثرها على البيئة. إن دراسة النماذج الخاصة في معرفة تأثير عملية تصفية المياه المالحة على عمق قاع البحر تتم بواسطة استخدام معادلة الانتشار والاستناد في بعدين. ولتوضيح هذه المسألة يتم دراسة حالة الشاطئ البسيط المنتظم الانخفادات، وحالة الشاطئ العمودي ذو عمق ثابت و الشاطئ المتدرج والمتغير في عمقه.

ويتم وصف الحلول بالرسومات البيانية لمحتويات تركيز الملوحة والتي تبين ما يلي: إذا كان مصب المياه المالحة قريب من الشاطئ فستجه الأملاح نحوه. ولكنه قد يتبع وبدون بناء مصب بعيد المدى إمكانية تخفيف درجة الملوحة بإنشاء مدرج متقطع في عمق قاع البحر.