Kuwait J. Sci. **41** (2) pp. 65-83, 2014

Beltrami-Meusnier formulas of generalized semi ruled surfaces in semi Euclidean space

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ABSTRACT

In this paper, we study the sectional curvatures of the generalized semi ruled surfaces in semi Euclidean space E_v^{n+1}. The first fundamental form and the metric coefficients of the generalized semi ruled surfaces are calculated and in these regards, Riemannian-Christoffel curvatures are obtained by the help of Christoffel Symbols. So, the curvatures of arbitrary non-degenerate tangential sections of the generalized semi ruled surface are investigated. In addition to this, the relations between the sectional curvatures are obtained. These are called semi Euclidean Beltrami-Meusnier formulas.

Keywords: Semi Euclidean space; ruled surface; sectional curvature; Beltrami-Meusnier formula.

1. PRELIMINARIES

E_v^{n+1}semi Euclidean space is an Euclidean space provided with the metric tensor

$$\left\langle \vec{X}, \vec{Y} \right\rangle = -\sum_{i=1}^{\nu} x_i y_i + \sum_{i=\nu+1}^{n+1} x_i y_i \qquad (1.1)$$

where $\overrightarrow{X} = (x_1, ..., x_n, x_{n+1})$ and $\overrightarrow{Y} = (y_1, ..., y_n, y_{n+1})$. Especially, if $v = 0$, then E_0^{n+1} is called Euclidean space, if $v = 1 (n \geq 2)$ then E_v^{n+1} is called as Minkowski space, (O'Neill, 1983). Since $\langle \; , \; \rangle$ is an indefinite metric, recall that a vector $\overrightarrow{X} \in E_v^{n+1}$ can have one of the three causal characters; it can be spacelike if $\left\langle \vec{X}, \vec{X} \right\rangle > 0$ or $\vec{X} = 0$, timelike if $\left\langle \vec{X}, \vec{X} \right\rangle < 0$ and null (lightlike) if $\left\langle \vec{X}, \vec{X} \right\rangle = 0$ and $\vec{X} \neq 0$. Similarly, an arbitrary curve $\alpha = \alpha(t) \subset E_v^{n+1}$ can locally be spacelike, timelike or null (lightlike) if all of its velocity vectors $\overset{\bullet}{\alpha}(t)$ are respectively spacelike, timelike or null (lightlike), (O'Neill, 1983). The norm of $\overrightarrow{X} \in E_v^{n+1}$ is given as $\left\|\overrightarrow{X}\right\| = \sqrt{\left|\left\langle \overrightarrow{X}, \overrightarrow{X} \right\rangle\right|}$. Let the set of all timelike vectors in E_v^{n+1} be Γ, we call

$$C\left(\overrightarrow{X}\right) = \left\{ \overrightarrow{Y} \in \Gamma \middle| \left\langle \overrightarrow{Y}, \overrightarrow{X} \right\rangle < 0 \right\} \tag{1.2}$$

for $\overrightarrow{X} \in \Gamma$, as time-cone of semi Euclidean space E_v^{n+1} including vector $\overrightarrow{X}$, (O'Neill, 1983).

If the timelike vectors $\vec{X}$ and $\vec{Y}$ are in the same time-cone in E_v^{n+1}, then there is a unique non-negative real number such that

$$\left\langle \vec{X}, \vec{Y} \right\rangle = -\|\vec{X}\|\|\vec{Y}\| \cosh\theta \tag{1.3}$$

where the number θ is called an angle between the timelike vectors, (O'Neill, 1983).

If $\overrightarrow{X}$ and $\overrightarrow{Y}$ are spacelike vectors in E_v^{n+1} that span a timelike subspace, there is unique positive real number such that

$$\left\langle \overrightarrow{X}, \overrightarrow{Y} \right\rangle = \left\|\overrightarrow{X}\right\|\left\|\overrightarrow{Y}\right\| \cosh\theta \tag{1.4}$$

where the semi Euclidean angle between $\overrightarrow{X}$ and $\overrightarrow{Y}$ is defined with θ, (Ratcliffe, 1994).

Suppose that $\overrightarrow{X}$ is spacelike vector and $\overrightarrow{Y}$ is timelike vector in E_v^{n+1}, Then, there is unique non-positive real number of such that

$$\left|\left\langle \overrightarrow{X}, \overrightarrow{Y} \right\rangle\right| = \left\|\overrightarrow{X}\right\|\left\|\overrightarrow{Y}\right\| \sinh\theta \tag{1.5}$$

where θ is a semi Euclidean angle between the $\overrightarrow{X}$ and $\overrightarrow{Y}$, (Ratcliffe, 1994).

A two-dimensional subspace Π of the tangent space $T_p(M)$ is called a tangent plane to M at p. For tangent vectors v, w define $Q(\vec{v}, \vec{w}) = \langle \vec{v}, \vec{v} \rangle \langle \vec{w}, \vec{w} \rangle - \langle \vec{v}, \vec{w} \rangle^2$.Tangent plane Π is non-degenerate if and only if $Q(\vec{v}, \vec{w}) \neq 0$ for one basis $\vec{v}, \vec{w}$ of Π. The absolute value $|Q(\vec{v}, \vec{w})|$ is the square of the area of the parallelogram with sides $\vec{v}$ and$\vec{w}$. $Q(\vec{v}, \vec{w})$ is positive if $g|\,\Pi$ is definite, negative if it is indefinite (use an orthonormal basis), (O'Neill, 1983).

Lemma 1.1. *Let Π be a non-degenerate tangent plane to M to p. The number*

$$K(\vec{v}, \vec{w}) = \frac{\langle R_{\vec{v}\vec{w}}\vec{v}, \vec{w} \rangle}{Q(\vec{v}, \vec{w})} \tag{1.6}$$

is independent of the choice of basis $\vec{v}, \vec{w}$ *for* Π, *and is called the sectional curvature* $K(\Pi)$ *of* Π. *Here* R *is a* $(1,3)-$*tensor field on* M *called Riemannian curvature tensor of* M, *(O'Neill, 1983).*

2. GENERALIZED SEMI RULED SURFACES IN E_v^{n+1}

After recalling some usual definitions and notations from algebra of the semi Euclidean space, we investigate some relations for the sectional curvatures of generalized semi ruled surfaces introduced by Ekici & Görgülü (2000). Generalized ruled surface theory in $n-$ dimensional Euclidean space E^n were studied widely in the last century in (Thas, 1978; Frank & Giering, 1976, 1978). Frank & Giering (1979) obtained the Beltrami Euler and Beltrami Meusnier formulas by applying the well known theorems concerning the curvatures in the classical surface theory to the tangential sections. The sectional curvatures of generalized timelike ruled surfaces in $n-$ dimensional Minkowski space were investigated by (Ersoy & Tosun, 2009, 2010) and Lorentzian Beltrami-Meusnier formula was obtained in (Ersoy & Tosun, 2013) which were inspired by (Tosun & Kuruoğlu, 1998). In these regards, to introduce generalized expressions for the sectional curvatures of generalized semi ruled surfaces, let us investigate the first fundamental form, the metric coefficients of these surfaces and the components of Riemann Christoffel curvature by using Christoffel symbols.

Let $\{e_1(t), e_2(t), \ldots, e_k(t)\}$, $k < n$,be an orthonormal vector field which is defined at the each point $\alpha(t)$ of a non-null curve in $(n+1)-$dimensional semi Euclidean space E_v^{n+1}. This system is denoted by $E_{k,\mu}(t)$ and is given by

$$E_{k,\mu}(t) = Sp\{e_1(t), e_2(t), \ldots, e_k(t)\} \quad , \quad 0 \leq \mu \leq v,$$

where

$$\langle e_i(t), e_j(t) \rangle = \varepsilon_i \delta_{ij} \quad , \qquad \varepsilon_i = \begin{cases} \quad 1 \leq i \leq k-\mu \\ -1 \quad , \ k-\mu+1 \leq i \leq k. \end{cases}$$

For $\mu \geq 1$, it is clear that the subspace $E_{k,\mu}(t)$ has number of μ timelike vectors. $E_{k,0}(t) = E_k(t)$ is an Euclidean subspace if there is no timelike vector in $E_{k,\mu}(t)$, that is $\mu = 0$. If there is one timelike vector (that is, $\mu = 1$), then $E_{k,1}(t)$ is Minkowski subspace. Thus, in semi Euclidean space $E_v^{n+1}(t)$ a generalized semi ruled surface (semi ruled surface) is given parametrically by

$$\varphi(t, u_1, u_2, \ldots, u_k) = \alpha(t) + \sum_{i=1}^{k} u_i e_i(t) \tag{2.1}$$

and denoted by M, (Ekici & Görgülü, 2000).

Here, the subspaces $E_{k,\mu}(t)$ and the curve α are called the generating spaces and base curve, respectively. We assume that

$$\left\{ \overset{\bullet}{\alpha}(t) + \sum_{i=1}^{k} u_i \underset{i}{\overset{\bullet}{e}}(t), e_1(t), ..., e_k(t) \right\} \tag{2.2}$$

is linearly independent. The subspace

$$A(t) = Sp\left\{ e_1(t), ..., e_k(t), \underset{1}{\overset{\bullet}{e}}(t), ..., \underset{k}{\overset{\bullet}{e}}(t) \right\} \tag{2.3}$$

is called asymptotic bundle of M with respect to $E_{k,\mu}(t)$ such that $\underset{i}{\overset{\bullet}{e}}(t)$ is the derivative of the vector field of $e_i(t)$, $1 \leq i \leq k$, (Ekici & Görgülü, 2000).

If $\dim A(t) = k + m$, $0 \leq m \leq k$, then there exists an orthonormal basis $\{e_1(t), ..., e_k(t), a_{k+1}(t), ..., a_{k+m}(t)\}$ of $A(t)$ containing $E_{k,\mu}(t)$. Also, for the orthonormal base $\{e_1(t), e_2(t), ..., e_k(t)\}$, there are following equations

$$\begin{aligned} \overset{\bullet}{e_i} &= \textstyle\sum_{j=1}^{k} \alpha_{ij}\, e_j + \varepsilon_{k+i}\, \kappa_i\, a_{k+i} \quad , \quad 1 \leq i \leq m \\ \overset{\bullet}{e}_h &= \textstyle\sum_{j=1}^{h} \alpha_{hj}\, e_j \quad , \quad m+1 \leq h \leq k \end{aligned} \tag{2.4}$$

where

$$\varepsilon_{ij}\alpha_{ij} = -\alpha_{ji} \ , \ \ \varepsilon_j = \langle e_j, e_j \rangle \ , \ \ \varepsilon_{ij} = \varepsilon_i \varepsilon_j \tag{2.5}$$

and

$$\begin{aligned} &\kappa_1 > \kappa_2 > ... > \kappa_{m-r} > 0 \\ &\kappa_{m-r+1} < \kappa_{m-r+2} < ... < \kappa_m < 0, \end{aligned} \tag{2.6}$$

for $r \leq \mu$, (Ekici & Görgülü, 2000). The subspace

$$T(t) = Sp\left\{ e_1(t), ..., e_k(t), \overset{\bullet}{e}_1(t), ..., \overset{\bullet}{e}_k(t), \overset{\bullet}{\alpha}(t) \right\} \tag{2.7}$$

is called tangential sub-bundle of M with respect to $E_{k,\mu}(t)$. Thus,

$$k + m \leq \dim T(t) \leq k + m + 1 \quad , \quad 0 \leq m \leq k.$$

Suppose that $\dim T(t) = k + m$, $0 \leq m \leq k$. Then $\{e_1, ..., e_k, a_{k+1}, ..., a_{k+m}\}$ is an orthonormal basis both of asymptotic bundle $A(t)$ and tangential bundle

$T(t)$. This means that $A(t)$ is coincident with $T(t)$. Assume that, for all $t, \dim T(t) = k + m + 1,\ 0 \le m \le k$. Thus, one can find an orthonormal basis for $T(t)$ as $\{e_1, ..., e_k, a_{k+1}, ..., a_{k+m}, a_{k+m+1}\}$. In the case of $\dim T(t) = k + m + 1$, $(k + 1)-$ dimensional generalized semi ruled surface M has a $(k - m)-$dimensional tangent subspace called central space of M at the each point $\alpha(t)$ and is denoted by $Z_{k-m,r}(t) \subset E_{k,\mu}(t)$. While the semi subspace $Z_{k-m,r}(t)$ is moving through the base curve α of M, it generates a $(k - m + 1)-$dimensional ruled surface contained by M which is called as $(k - m + 1)-$dimensional central ruled surface. This surface is denoted by Ω. Moreover, the central surface Ω is also, a semi ruled surface because $Z_{k-m,r}(t)$ is a semi subspace, (Ekici & Görgülü, 2000).

We assume that the base curve of M is also the base curve of $\Omega \subset M$. Therefore, we have

$$\dot{\alpha}(t) = \sum_{\nu=1}^{k} \zeta_\nu e_\nu + \eta_{m+1} a_{k+m+1} \quad , \quad \eta_{m+1} \neq 0. \tag{2.8}$$

The tangential space $T(t)$ of M is perpendicular to the asymptotic bundle $A(t)$ at the central points. If we consider the equation (2.8) at the central point of central ruled surface $\Omega \subset M$, then we get

$$u_\sigma = 0 \quad , \quad 1 \le \sigma \le m. \tag{2.9}$$

Considering the equations (2.4) and (2.8), if we differentiate the equation (2.1) with respect to t and u_i, $1 \le i \le m$, we get

$$\varphi_t = \sum_{i=1}^{k} \left(\zeta_i + \sum_{j=1}^{k} u_j \alpha_{ji} \right) e_i + \textstyle\sum_{\sigma=1}^{m} \varepsilon_{k+\sigma} u_\sigma \kappa_\sigma a_{k+\sigma} + \eta_{m+1} a_{k+m+1},$$

$$\varphi_{u_i} = e_i, \quad 1 \le i \le k,$$

respectively. Thus, the canonical base of the tangential bundle of generalized semi ruled surface M is

$$\left\{ \sum_{i=1}^{k} \left(\zeta_i + \sum_{j=1}^{k} u_j \alpha_{ji} \right) e_i + \sum_{\sigma=1}^{m} \varepsilon_{k+\sigma} u_\sigma \kappa_\sigma a_{k+\sigma} + \eta_{m+1} a_{k+m+1},\ e_1,\ e_2, \ldots, e_k \right\}. \tag{2.10}$$

Now, we can evaluate the first fundamental form of M and the metric coefficients with respect to this canonical base. In conventional notation we choose $u_0 = t$ and calculate the metric coefficients of M as follows:

$$\begin{aligned}
g_{00} &= \langle \varphi_t\,,\ \varphi_t\rangle = \sum_{i=1}^{k}\varepsilon_i\left(\zeta_i+\sum_{j=1}^{k}u_j\alpha_{ji}\right)^2+\sum_{\sigma=1}^{m}\varepsilon_{k+\sigma}(u_\sigma\kappa_\sigma)^2+\varepsilon_{k+m+1}(\eta_{m+1})^2,\\
g_{i0} &= \left\langle \varphi_{u_i},\ \varphi_t\right\rangle = \varepsilon_i\left(\zeta_i+\sum_{j=1}^{k}u_j\alpha_{ji}\right),\quad 1\le i\le k,\\
g_{ij} &= \left\langle \varphi_{u_j},\varphi_{u_j}\right\rangle = \varepsilon_i\delta_{ij}\quad 1\le i,j\le k.
\end{aligned}\tag{2.11}$$

Therefore, the determinant of the matrix of the first fundamental form of M is as follows:

$$g=\det[g_{ab}]=\varepsilon\left(\sum_{\sigma=1}^{m}\varepsilon_{k+\sigma}(u_\sigma\kappa_\sigma)^2+\varepsilon_{k+m+1}(\eta_{m+1})^2\right)\quad,\qquad 1\le a,b\le k \tag{2.12}$$

where $\varepsilon=\varepsilon_1.\varepsilon_2.\ \dots\ .\varepsilon_k$. Since g is non-degenerate at the each point of $T_M(p)$ the matrix $[g_{ab}]$ is invertible and the inverse matrix is denoted by $\left[\boldsymbol{g}^{\boldsymbol{ab}}\right]$. The coefficients of the inverse matrix $\left[\boldsymbol{g}^{\boldsymbol{ab}}\right]$ are as follows:

$$\begin{aligned}
g^{00} &= \varepsilon g^{-1}\ ,\\
g^{i0} &= -\varepsilon g^{-1}\left(\zeta_i+\sum_{j=1}^{k}u_j\alpha_{ji}\right)\qquad ,1\le i\le k,\\
g^{i\lambda} &= g^{-1}\left(\left(\zeta_i+\sum_{j=1}^{k}u_j\alpha_{ji}\right)\left(\zeta_\lambda+\sum_{j=1}^{k}u_j\alpha_{j\lambda}\right)\varepsilon+\delta_{i\lambda}\varepsilon_i g\right)\qquad ,\quad 1\le i,\lambda\le k.
\end{aligned}\tag{2.13}$$

From (Beem *et al.,* 1981), the Koszul equation is

$$\Gamma_{ij}^{k}=\frac{1}{2}\sum_{m}g^{km}\left[\frac{\partial g_{jm}}{\partial x_i}+\frac{\partial g_{im}}{\partial x_j}-\frac{\partial g_{ij}}{\partial x_m}\right].$$

If we substitute the equations (2.11) and (2.13) into the equation (2.14), then we reach the Christoffel symbols for $1\le i,j,\lambda\le k$, as follows:

$$\Gamma_{00}^{0} = \frac{1}{2g}\left[\frac{\partial g}{\partial u_0} + \sum_{i=1}^{k}\left(\zeta_i + \sum_{j=1}^{k} u_j \alpha_{ji}\right)\frac{\partial g}{\partial u_i}\right],$$

$$\Gamma_{00}^{\lambda} = -\frac{1}{2g}\left(\zeta_\lambda + \sum_{j=1}^{k} u_j \alpha_{j\lambda}\right)\left(\frac{\partial g}{\partial u_0} + \sum_{i=1}^{k}\left(\zeta_i + \sum_{j=1}^{k} u_j \alpha_{ji}\right)\frac{\partial g}{\partial u_i}\right)$$
$$+\left(\left(\underset{\lambda}{\dot{\zeta}} + \sum_{j=1}^{k} u_j \underset{j\lambda}{\dot{\alpha}}\right) + \sum_{i=1}^{k}\left(\zeta_i + \sum_{j=1}^{k} u_j \alpha_{ji}\right)\alpha_{i\lambda} - \varepsilon\varepsilon_\lambda \frac{1}{2}\frac{\partial g}{\partial u_\lambda}\right),$$

$$\Gamma_{ij}^{0} = \Gamma_{ji}^{0} = 0, \tag{2.15}$$

$$\Gamma_{ij}^{\lambda} = \Gamma_{ji}^{\lambda} = 0,$$

$$\Gamma_{\lambda 0}^{0} = \Gamma_{0\lambda}^{0} = \frac{1}{2g}\frac{\partial g}{\partial u_\lambda},$$

$$\Gamma_{i0}^{\lambda} = \Gamma_{0i}^{\lambda} = \frac{1}{2g}\left[-\left(\zeta_\lambda + \sum_{j=1}^{k} u_j \alpha_{j\lambda}\right)\frac{\partial g}{\partial u_i} + 2g(\alpha_{i\lambda})\right].$$

where $\varepsilon = \varepsilon_1 . \varepsilon_2 . \varepsilon_3 . \ \ldots \ .\varepsilon_k$ and $\varepsilon_i = \mp 1$.

The Riemannian-Christoffel curvature tensor of M is given by

$$R_{hlij} = \sum_{r=0}^{k} g_{rh}\left(\frac{\partial}{\partial u_i}\Gamma_{jl}^{r} - \frac{\partial}{\partial u_j}\Gamma_{il}^{r} - \sum_{s=0}^{k}\Gamma_{il}^{s}\Gamma_{js}^{r} + \sum_{s=0}^{k}\Gamma_{jl}^{s}\Gamma_{is}^{r}\right).$$

Considering the equations (2.11) and (2.15), the Riemannian-Christoffel curvature tensors are

$$R_{0000} = R_{i000} = R_{ij00} = 0, 1 \le i,j \le k, \tag{2.17}$$

$$R_{0hij} = 0, 1 \le i,j,h \le k, \tag{2.18}$$

$$R_{lhij} = 0, 1 \le i,j,h,l \le k, \tag{2.19}$$

$$R_{i0j0} = \varepsilon\left(-\frac{1}{2}\frac{\partial^2 g}{\partial u_i \partial u_j} + \frac{1}{4g}\frac{\partial g}{\partial u_i}\frac{\partial g}{\partial u_j}\right). \tag{2.20}$$

Since a normal tangent vector n of the generalized semi ruled surface M will be orthogonal to $E_{k,\mu}(t)$ at the each point $\xi(t, u_\nu)$, it is defined to be

$$n=\sum_{\sigma=1}^{m}\varepsilon_{k+\sigma}u_{\sigma}\kappa_{\sigma}(t)a_{k+\sigma}(t)+\eta_{m+1}a_{k+m+1}(t)\quad,\quad \eta_{m+1}\neq 0 \tag{2.21}$$

where n is either a timelike or spacelike vector.

Thus, the following theorem can be given related to the principal sectional curvatures at the point $\xi\in M$.

Theorem 2.1. *Let M be a generalized semi ruled surface with the central ruled surface in E_v^{n+1} and n be a non-null normal tangent vector of M orthogonal to $E_{k,\mu}(t)$. Thus, the i^{th} principal sectional curvature (e_i,n) at the each point $\xi\in M$ is*

$$K_{\xi}(e_i,n)=\varepsilon_i\left(-\frac{1}{2g}\frac{\partial^2 g}{\partial u_i^2}+\frac{1}{4g^2}\left(\frac{\partial g}{\partial u_i}\right)^2\right),\quad 1\leq i\leq k. \tag{2.22}$$

Let $e(t)$ be a unit vector in the generating space $E_{k,\mu}(t)$, that is, we write

$$e(t)\in Sp\{e_1(t),\,\ldots,\,e_m(t),e_{m+1}(t),\,\ldots,\,e_k(t)\} \tag{2.23}$$

and $e(t)$ is either unit spacelike or unit timelike vector. Now, we investigate these situations, separately.

(i) Let $e(t)$ be spacelike vector. In this case, we write

$$e=\sum_{x=1}^{s}\sinh\theta_x e_x+\sum_{y=s+1}^{m}\cosh\theta_y e_y+\sum_{z=m+1}^{m+\mu-s}\sinh\theta_z e_z+\sum_{w=m+1+\mu-s}^{k}\cosh\theta_w e_w \tag{2.24}$$

and

$$-\sum_{x=1}^{s}\sinh^2\theta_x+\sum_{y=s+1}^{m}\cosh^2\theta_y-\sum_{z=m+1}^{m+\mu-s}\sinh^2\theta_z+\sum_{w=m+1+\mu-s}^{k}\cosh^2\theta_w=1 \tag{2.25}$$

where the hyperbolic angles $\theta_1,\,\theta_2,\,\ldots,\,\theta_s,\,\theta_{s+1},\,\ldots,\,\theta_m,\,\theta_{m+1},\,\ldots,\,\theta_k$ are the angles between spacelike unit vector e and the base vectors $e_1,e_2,\ldots,e_s,e_{s+1},\ldots,e_m,e_{m+1},\ldots,e_k$, respectively.

(ii) Let $e(t)$ be a timelike vector. Therefore, we have

$$e=\sum_{x=1}^{s}\cosh\theta_x e_x+\sum_{y=s+1}^{m}\sinh\theta_y e_y+\sum_{z=m+1}^{m+\mu-s}\cosh\theta_z e_z+\sum_{w=m+1+\mu-s}^{k}\sinh\theta_w e_w \tag{2.26}$$

and

$$-\sum_{x=1}^{s}\cosh^2\theta_x+\sum_{y=s+1}^{m}\sinh^2\theta_y-\sum_{z=m+1}^{m+\mu-s}\cosh^2\theta_z+\sum_{w=m+1+\mu-s}^{k}\sinh^2\theta_w=-1 \quad (2.27)$$

where the hyperbolic angles between the base vectors $e_1, e_2, \ldots, e_s, e_{s+1}, \ldots, e_m, e_{m+1}, \ldots, e_k$ and timelike unit vector e are $\theta_1, \theta_2, \ldots, \theta_s, \theta_{s+1}, \ldots, \theta_m, \theta_{m+1}, \ldots, \theta_k$, respectively.

Considering these situations, we give following theorem.

Theorem 2.2. *Let M be a semi ruled surface with the central ruled surface in $E_v^{n+1}$and n be the normal tangent vector orthogonal to $E_{k,\mu}(t)$ of M. In this case, there exist the following relation between the sectional curvature of the non-degenerate section (e,n) and the principal sectional curvatures at the point $\zeta \in \Omega \subset M$ as follows:*

(i) *If the unit vector $e(t)$ is a spacelike vector, then*

$$K_\zeta(e,n)=-\sum_{i=1}^{s}\sinh^2\theta_i K_\zeta(e_i,n)+\sum_{j=s+1}^{m}\cosh^2\theta_j K_\zeta(e_j,n), \quad (2.28)$$

(ii) *If the unit vector $e(t)$ is a timelike vector, then*

$$K_\zeta(e,n)=\sum_{i=1}^{s}\cosh^2\theta_i K_\zeta(e_i,n)-\sum_{j=s+1}^{m}\sinh^2\theta_j K_\zeta(e_j,n) \quad (2.29)$$

where the angles $\theta_1, \theta_2, \ldots, \theta_s, \theta_{s+1}, \ldots, \theta_m, \theta_{m+1}, \ldots, \theta_k$ are the hyperbolic angles between the unit vector e (spacelike or timelike) and the base vectors $e_1, e_2, \ldots, e_s, e_{s+1}, \ldots, e_m, e_{m+1}, \ldots, e_k$, respectively.

3. BELTRAMI-MEUSNIER FORMULAS OF SEMI EUCLIDEAN SPACE E_v^{n+1}

Let Ω be a central ruled surface of the generalized semi ruled surface M in E_v^{n+1}. Therefore, any unit vector at the each central point $\zeta \in \Omega$ is defined to be

$$\begin{aligned} a=\lambda_0\frac{n}{\|n\|}+\lambda_1e_1+\ldots+\lambda_{s-1}e_{s-1}+\lambda_se_s+\lambda_{s+1}e_{s+1}+\ldots+\lambda_me_m \\ +\lambda_{m+1}e_{m+1}+\ldots+\lambda_{m+\mu-s}e_{m+\mu-s}+\lambda_{m+1+\mu-s}e_{m+1+\mu-s}+\ldots+\lambda_ke_k \end{aligned}, \langle a,a\rangle=\pm1,$$

where a and e_σ, $1\le\sigma\le m$, are linearly independent. The unit vector a is either spacelike or timelike vector. There exist the following four cases depending on whether the normal tangent vector n which is orthogonal to the generating space $E_{k,\mu}(t)$ of M is spacelike or timelike vector.

1) The unit vector a and the unit normal tangent vector n are spacelike.
2) The unit vector a is a spacelike and the unit normal tangent vector n is timelike.
3) The unit vector a is a timelike and the unit normal tangent vector n is spacelike.
4) The unit vector a and the unit normal tangent vector n are timelike.

Now, we investigate these situations, separately.

1) Let the unit vector a and the unit normal tangent vector n be spacelike vectors. In this case, any spacelike vector a at the central point $\xi \in \Omega$ can be written as follows

$$a = \cosh\psi_0 \frac{n}{\|n\|} + \sum_{x=1}^{s} \sinh\psi_x e_x + \sum_{y=s+1}^{m} \cosh\psi_y e_y + \sum_{z=m+1}^{m+\mu-s} \sinh\psi_z e_z + \sum_{w=m+1+\mu-s}^{k} \cosh\psi_w e_w \quad (3.1)$$

and

$$-\sum_{x=1}^{s} \sinh^2\psi_x + \sum_{y=s+1}^{m} \cosh^2\psi_y - \sum_{z=m+1}^{m+\mu-s} \sinh^2\psi_z + \sum_{w=m+1+\mu-s}^{k} \cosh^2\psi_w = 1 \quad (3.2)$$

where e_s, $1 \leq s \leq m$, is a timelike vector and the angles $\psi_0, \psi_1, \ldots, \psi_s, \ldots, \psi_k$ are the hyperbolic angles between the spacelike unit vector a and the vectors $n, e_1, \ldots, e_s, \ldots, e_k$, respectively.

2) Let the unit vector a be a spacelike vector and the unit normal tangent vector n be a timelike vector. Suppose that e_s, $1 \leq s \leq m$, is a timelike vector. In this case, we write the spacelike unit vector a at the central point $\xi \in \Omega$ as

$$a = \sinh\psi_0 \frac{n}{\|n\|} + \sum_{x=1}^{s} \sinh\psi_x e_x + \sum_{y=s+1}^{m} \cosh\psi_y e_y + \sum_{z=m+1}^{m+\mu-s} \sinh\psi_z e_z + \sum_{w=m+1+\mu-s}^{k} \cosh\psi_w e_w \quad (3.2)$$

and

$$-\sum_{x=1}^{s} \sinh^2\psi_x + \sum_{y=s+1}^{m} \cosh^2\psi_y - \sum_{z=m+1}^{m+\mu-s} \sinh^2\psi_z + \sum_{w=m+1+\mu-s}^{k} \cosh^2\psi_w = 1.$$

So that the angles $\psi_0, \psi_1, \ldots, \psi_s, \ldots, \psi_k$ are the hyperbolic angles between the spacelike unit vector a and the vectors $n, e_1, \ldots, e_s, \ldots, e_k$, respectively.

3) Let the unit vector a be a timelike vector and the unit normal tangent vector n be a spacelike vector. As above cases, at the central point $\xi \in \Omega$ any timelike vector a is written as follows:

$$a = \sinh \psi_0 \frac{n}{\|n\|} + \sum_{x=1}^{s} \cosh \psi_x e_x + \sum_{y=s+1}^{m} \sinh \psi_y e_y + \sum_{z=m+1}^{m+\mu-s} \cosh \psi_z e_z + \sum_{w=m+1+\mu-s}^{k} \sinh \psi_w e_w. \quad (3.3)$$

It is clear that

$$-\sum_{x=1}^{s} \cosh^2 \psi_x + \sum_{y=s+1}^{m} \sinh^2 \psi_y - \sum_{z=m+1}^{m+\mu-s} \cosh^2 \psi_z + \sum_{w=m+1+\mu-s}^{k} \sinh^2 \psi_w = -1$$

where the angles $\psi_0, \psi_1, \ldots, \psi_s, \ldots, \psi_k$ are the hyperbolic angles between the timelike unit vector a and the vectors $n, e_1, \ldots, e_s, \ldots, e_k$, respectively. In addition to that, the vectors e_s, $1 \leq s \leq m$,are the timelike vectors.

4) Let the unit vector a and the unit normal tangent vector n be timelike vectors. Let e_s, $1 \leq s \leq m$, be timelike vectors. In this case, timelike unit vector a can be written as follows:

$$a = \cosh \psi_0 \frac{n}{\|n\|} + \sum_{x=1}^{s} \cosh \psi_x e_x + \sum_{y=s+1}^{m} \sinh \psi_y e_y + \sum_{z=m+1}^{m+\mu-s} \cosh \psi_z e_z + \sum_{w=m+1+\mu-s}^{k} \sinh \psi_w e_w. \quad (3.4)$$

In addition, there is

$$-\sum_{x=1}^{s} \cosh^2 \psi_x + \sum_{y=s+1}^{m} \sinh^2 \psi_y - \sum_{z=m+1}^{m+\mu-s} \cosh^2 \psi_z + \sum_{w=m+1+\mu-s}^{k} \sinh^2 \psi_w = -1.$$

where the hyperbolic angles between the timelike unit vector a and the vectors $n, e_1, \ldots, e_s, \ldots, e_k$ are $\psi_0, \psi_1, \ldots, \psi_s, \ldots, \psi_k$, respectively.

Considering equation (1.6), at the point $(\zeta + ue_\sigma) \in M$, the curvature of non-degenerate section (e_σ, a) is

$$K_{\zeta+ue_\sigma}(e_\sigma, a) = \frac{\beta_\sigma \beta_\sigma \lambda_0 \lambda_0 R_{\sigma 0 \sigma 0}}{\langle e_\sigma, e_\sigma \rangle \langle a, a \rangle - \langle e_\sigma, a \rangle^2} \quad , \quad 1 \leq \sigma \leq m. \quad (3.5)$$

Therefore, taking the cases 1), 2), 3) and 4) into consideration, we give the following theorem below about the relationship between the curvatures of section (e_x, a), $1 \leq x \leq s$ and the x^{th} principal section (e_x, n).

Theorem 3.1. *Let M be a generalized semi ruled surface with the central ruled surface in E_v^{n+1} and the unit vector n be a non-null normal tangent vector of M orthogonal to the generating space $E_{k,\mu}(t)$. In this case, the following relations between the curvatures of non-degenerate section (e_x, a), $1 \leq x \leq s$, and non-degenerate the x^{th} principal section (e_x, n) at the point $(\zeta + ue_x) \in M$ exist:*

(i) *If the unit vector a and the unit normal tangent vector n are spacelike vectors, then*

$$\left(1 + \sinh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = \cosh^2 \psi_0 K_{\zeta+ue_x}(e_x, n) \tag{3.6}$$

(ii) *If the unit vector a is a spacelike vector and the unit normal tangent vector n is timelike vector, then*

$$\left(1 + \sinh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = -\sinh^2 \psi_0 K_{\zeta+ue_x}(e_x, n), \tag{3.7}$$

(iii) *If the unit vector a is a timelike vector and the unit normal tangent vector n is spacelike vector, then*

$$\left(1 - \cosh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = -\sinh^2 \psi_0 K_{\zeta+ue_x}(e_x, n), \tag{3.8}$$

(iv) *If the unit vector a and the unit normal tangent vector n are timelike vectors, then*

$$\left(1 - \cosh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = \cosh^2 \psi_0 K_{\zeta+ue_x}(e_x, n) \tag{3.9}$$

where the base vector e_x are a timelike vectors and the non-null unit vector a is linearly independent of the timelike base vector e_x at the each point $\xi \in \Omega$. In addition to that, the hyperbolic angles are ψ_0 and ψ_x between a and n, and between a and e_x, respectively.

Proof. Let the base vectors e_x, $1 \leq x \leq s$, be the timelike vectors in the generating space $E_{k,\mu}(t)$ of the generalized semi ruled surface M in E_v^{n+1} and the non-null unit vector a be linearly independent of the timelike base vectors e_x at the each point $\xi \in \Omega$.

(i) Suppose that the unit vector a and the unit normal tangent vector n are spacelike vectors. Let the coordinates of the base vectors e_i, $1 \leq i \leq k$, and the spacelike unit vector a (given by equation (3.1)) be $(\beta_0, \beta_1, \ldots, \beta_i, \ldots, \beta_k)$ and $(\gamma_0, \gamma_1, \ldots, \gamma_s, \ldots, \gamma_k)$, respectively. In this case, we can write

$$\beta_0 = \langle e_i, e_0 \rangle = 0 \quad , \quad \beta_i = \langle e_i, e_i \rangle = \varepsilon_i \quad , \quad 1 \leq i \leq k \ ,$$

and

$$\gamma_0 = \langle a, e_0 \rangle = \frac{\cosh \psi_0}{\|n\|} \quad , \quad \gamma_x = \langle a, e_x \rangle = \sinh \psi_x \quad , \quad 1 \leq x \leq s \quad ,$$
$$\gamma_y = \langle a, e_y \rangle = \cosh \psi_y \quad , \quad s+1 \leq y \leq m.$$

Substituting the last equations and the equation (2.20) into the equation (3.5), we find

$$K_{\zeta+ue_x}(e_x, a) = \frac{\varepsilon_x^2 \dfrac{\cosh^2 \psi_0}{\|n\|^2} \; \varepsilon \left(-\dfrac{1}{2} \dfrac{\partial^2 g}{\partial u_x^2} + \dfrac{1}{4g} \left(\dfrac{\partial g}{\partial u_x} \right)^2 \right)}{\varepsilon_x - \sinh^2 \psi_x} \tag{3.10}$$

where $\varepsilon = \varepsilon_1 . \varepsilon_2 . \ ... \ .\varepsilon_k$. Since $\|n\|^2 = \varepsilon g$ and $\varepsilon_x = \langle e_x \, , \, e_x \rangle = -1$, we get

$$\left(1 + \sinh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = -\cosh^2 \psi_0 \left(-\frac{1}{2g} \frac{\partial^2 g}{\partial u_x^2} + \frac{1}{4g^2} \left(\frac{\partial g}{\partial u_x} \right)^2 \right). \tag{3.11}$$

From the equations (2.22) and (3.11), we find the relation between the curvature of non-degenerate section (e_x, a) and non-degenerate the x^{th} principal section (e_x, n) as

$$\left(1 + \sinh^2 \psi_x\right) K_{\zeta+ue_x}(e_x, a) = -\cosh^2 \psi_0 \varepsilon_x K_{\zeta+ue_x}(e_x, n).$$

To prove the (ii), (iii) and (iv), in a similar manner using the equations (3.2), (3.3), (3.4) and (2.20), (3.5), we reach the relations (3.7), (3.8) and (3.9), respectively. These complete the proof.

Now, considering the cases 1), 2), 3) and 4) for $s+1 \leq y \leq m$, we give the following theorem related to the relation between the curvatures of section (e_y, a) and the y^{th} principal section (e_y, n).

Theorem 3.2. *Let M be a generalized semi ruled surface with the central ruled surface in E_v^{n+1} and the unit vector n be a non-null normal tangent vector which is orthogonal to the generating space $E_{k,\mu}(t)$. In addition to that, the base vectors e_y,$s+1 \leq y \leq m$, are spacelike vectors and the non-null unit vector a is linearly independent of the spacelike base vectors e_y at the point $\xi \in \Omega$. Therefore, the following relationship between the curvatures of non-degenerate section (e_y, a) and non-degenerate the y^{th} principal section (e_y, n) at the point $(\zeta + ue_x) \in M$ as follows:*

(i) *Let the unit vector a and the unit normal tangent vector n be spacelike vectors, then*

$$\left(1-\cosh^2\psi_y\right)K_{\zeta+ue_y}\left(e_y,a\right)=\cosh^2\psi_0 K_{\zeta+ue_y}\left(e_y,n\right), \tag{3.12}$$

(ii) *Let the unit vector a be a spacelike vector and the unit normal tangent vector n be a timelike vector, therefore*

$$\left(1-\cosh^2\psi_y\right)K_{\zeta+ue_y}\left(e_y,a\right)=-\sinh^2\psi_0 K_{\zeta+ue_y}\left(e_y,n\right), \tag{3.13}$$

(iii) *Let the unit vector a be a timelike vector and the unit normal tangent vector n be a spacelike vector, in this case*

$$\left(1+\sinh^2\psi_y\right)K_{\zeta+ue_y}\left(e_y,a\right)=-\sinh^2\psi_0 K_{\zeta+ue_y}\left(e_y,n\right), \tag{3.14}$$

(iv) *Let the unit vector a and the unit normal tangent vector n be timelike vectors, then*

$$\left(1+\sinh^2\psi_\sigma\right)K_{\zeta+ue_\sigma}(e_\sigma,a)=\cosh^2\psi_0 K_{\zeta+ue_\sigma}(e_\sigma,n), \tag{3.15}$$

where the angles ψ_0 and ψ_y represent the hyperbolic angles a and n, and between a and e_y, respectively.

Proof. Let the base vectors e_y, $s+1\leq y\leq m$, be the spacelike vectors in $E_{k,\mu}(t)$ and the non-null unit vector a be linearly independent of the spacelike base vectors e_y at the each point $\xi\in\Omega$.

(i) Assume that the unit vector a and the unit normal tangent vector n are spacelike vectors. If the coordinates of the base vectors e_i, $1\leq i\leq k$ and spacelike unit vector a which is given by equation (3.1) are $(\beta_0,\beta_1,\ldots,\beta_i,\ldots,\beta_k)$ and $(\gamma_0,\gamma_1,\ldots,\gamma_s,\ldots,\gamma_k)$, respectively, then we have

$$\beta_0=\langle e_i,e_0\rangle=0 \quad , \quad \beta_i=\langle e_i,e_i\rangle=\varepsilon_i \quad , \quad 1\leq i\leq k \ ,$$

and

$$\gamma_0=\langle a,e_0\rangle=\frac{\cosh\psi_0}{\|n\|} \quad , \quad \gamma_x=\langle a,e_x\rangle=\sinh\psi_x \quad , \quad 1\leq x\leq s \quad ,$$
$$\gamma_y=\left\langle a,e_y\right\rangle=\cosh\psi_y \quad , \quad s+1\leq y\leq m.$$

Substituting these equations together with the equation (2.20) into the equation (3.5) and considering that $\|n\|^2=\varepsilon g$, we reach

$$\left(1-\cosh^2\psi_y\right)K_{\zeta+ue_y}\left(e_y,a\right)=\cosh^2\psi_0\left(-\frac{1}{2g}\frac{\partial^2 g}{\partial u_y^2}+\frac{1}{4g^2}\left(\frac{\partial g}{\partial u_y}\right)^2\right). \tag{3.16}$$

From the equations (2.22) and (3.16), we get the relation between curvature of non-degenerate section $\left(e_y,a\right)$ and non-degenerate the y^{th} principal section $\left(e_y,n\right)$, as

$$\left(1-\cosh^2\psi_y\right)K_{\zeta+ue_y}\left(e_y,a\right)=\cosh^2\psi_0\varepsilon_y K_{\zeta+ue_y}\left(e_y,n\right). \tag{3.17}$$

To prove the (ii), (iii) and (iv) in this theorem, in a similar way using the equations (3.2), (3.3), (3.4) together with the equations (2.20) and (3.5), we find the equations (3.13), (3.14) and (3.15), respectively.

Thus, the proof is completed.

Considering the unit vector e given by the equations (2.24) and (2.26) in $E_{k,\mu}(t)$ and the unit vector a given by the equations (3.1), (3.2), (3.3) and (3.4) at the point $\xi\in\Omega$, then we give the following theorem according to these situations of the vectors e and a, separately.

Theorem 3.3. *Let M be a generalized semi ruled surface with the central ruled surface in E_v^{n+1} and the vector n be a non-null normal tangent vector which is orthogonal to the generating space $E_{k,\mu}(t)$. Considering that the non-null vector a which is independent of the non-null unit vector e in $E_{k,\mu}(t)$ at the $\forall\zeta\in\Omega$. There exist the relations between the curvature of non-degenerate section (e,a) and the curvature of non-degenerate section (e,n) as follows:*

(i) Let the vectors n, e and a be spacelike vectors (timelike vectors). In this case

$$K_\zeta(e,a)=\frac{\cosh^2\psi_0}{1-\langle e,a\rangle^2}K_\zeta(e,n). \tag{3.18}$$

(ii) If the vectors n and a are spacelike vectors (timelike vectors), the vector e is timelike vector (spacelike vector), then

$$K_\zeta(e,a)=\frac{\cosh^2\psi_0}{1+\langle e,a\rangle^2}K_\zeta(e,n). \tag{3.19}$$

(iii) Let the vectors a and e be spacelike vectors (timelike vectors), the vector n be timelike vector (spacelike vector). Therefore

$$K_\zeta(e,a) = -\frac{\sinh^2\psi_0}{1-\langle e,a\rangle^2} K_\zeta(e,n). \tag{3.20}$$

(iv) If the vectors n and e are timelike vectors (spacelike vectors), the vector a is a spacelike vector (timelike vector). In this case

$$K_\zeta(e,a) = -\frac{\sinh^2\psi_0}{1+\langle e,a\rangle^2} K_\zeta(e,n). \tag{3.21}$$

where ψ_0 *is the hyperbolic angle between non-null unit vector* a *and non-null normal tangent vector* n.

Proof. Assume that the non-null unit vector e which is given by (2.24) and (2.26) in the generating space $E_{k,\mu}(t)$ and the non-null vector a given by the equations (3.1), (3.2), (3.3) and (3.4) which is linearly independent with the vector e at the point $\xi \in \Omega$. In addition to this, suppose that the non-null vector n is orthogonal to $E_{k,\mu}(t)$.

Considering the equation (1.6), the sectional curvature at the point $\xi \in \Omega$ is given by

$$K_\zeta(e,a) = \frac{\sum_{x=1}^{s} \beta_x\beta_x\lambda_0\lambda_0 R_{x0x0} + \sum_{y=s+1}^{m} \beta_y\beta_y\lambda_0\lambda_0 R_{y0y0}}{\langle e,e\rangle\langle a,a\rangle - \langle e,a\rangle^2}. \tag{3.22}$$

(i) First of all, let the vectors n, e and a be spacelike vectors. If the coordinates of e given by equation (2.24) and the tangent vector a given by equation (3.1) are $(\beta_0, \beta_1, \ldots, \beta_s, \ldots, \beta_k)$ and $(\gamma_0, \gamma_1, \ldots, \gamma_s, \ldots, \gamma_k)$, respectively. Then we write

$$\begin{aligned} &\beta_0 = \langle e, e_0\rangle = 0, \\ &\beta_x = \langle e, e_x\rangle = \sinh\theta_x \qquad l \le x \le s, \\ &\beta_y = \langle e, e_y\rangle = \cosh\theta_y \quad , \quad s+1 \le y \le m. \end{aligned} \tag{3.23}$$

and

$$\begin{aligned} &\gamma_0 = \langle a, e_0\rangle = \frac{\cosh\psi_0}{\|n\|} \\ &\gamma_x = \langle a, e_x\rangle = \sinh\psi_x \quad , \quad 1 \le x \le s \\ &\gamma_y = \langle a, e_y\rangle = \cosh\psi_y \quad , \quad s+1 \le y \le m. \end{aligned} \tag{3.24}$$

Substituting the equations (3.23), (3.24) and (2.2) into equation (3.22), we find

$$K_\zeta(e,a)=\frac{\frac{\cosh^2\psi_0}{\|n\|^2}\left(\sum_{x=1}^{s}\sinh^2\theta_x\left[\varepsilon\left(-\frac{1}{2}\frac{\partial^2 g}{\partial u_x^2}+\frac{1}{4g}\left(\frac{\partial g}{\partial u_x}\right)^2\right)\right]+\sum_{y=s+1}^{m}\cosh^2\theta_y\left[\varepsilon\left(-\frac{1}{2}\frac{\partial^2 g}{\partial u_y^2}+\frac{1}{4g}\left(\frac{\partial g}{\partial u_y}\right)^2\right)\right]\right)}{1-\langle e,a\rangle^2} \tag{3.25}$$

where $\varepsilon=\varepsilon_1.\varepsilon_2.\ \ldots\ .\varepsilon_k$. Using $\|n\|^2=-g$, from the equation (3.25), we reach

$$K_\zeta(e,a)=\frac{\cosh^2\psi_0\left(\sum_{x=1}^{s}\sinh^2\theta_x\left(-\frac{1}{2g}\frac{\partial^2 g}{\partial u_x^2}+\frac{1}{4g^2}\left(\frac{\partial g}{\partial u_x}\right)^2\right)+\sum_{y=s+1}^{m}\cosh^2\theta_y\left(-\frac{1}{2g}\frac{\partial^2 g}{\partial u_y^2}+\frac{1}{4g^2}\left(\partial g\partial u_y\right)^2\right)\right)}{1-\langle e,a\rangle^2}.$$

Considering $\varepsilon_x=-1$, $\varepsilon_y=1$and the equation (2.24) together with the last equation we complete the proof.

By following similar calculations and using the equations (2.28) and (2.29) together with the equations (2.22), (2.26), (3.1), (3.2), (3.3) and (3.4), we complete the proof of the other cases.

These relations given by the equations (3.18)-(3.21) between the curvature of the non-degenerate section (e,a) and the curvature of the non-degenerate section (e,n) are called **Semi Euclidean Beltrami Meusnier Formulas of generalized semi ruled surface at the central point $\xi\in\Omega$ in E_v^{n+1}.**

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Submitted : *20/05/2013*
Revised : *17/11/2013*
Accepted : *26/11/2013*

صيغ بلترامي – مونييه للسطوح المعممة شبه المسطرة في فضاء مثيل الاقليدي

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خلاصة

ندرس في هذا البحث المنحنى المقطعي للسطوح المعممة شبه المسطرة في فضاء مثيل أقليدي ونحصل في هذه الحالة على الصيغة الأساسية الأولى وعلى معاملات المقاس للسطوح المعممة المثيل اقليدية، كما نحصل كذلك على تقوسات ريمان – كريستوفل. كما نقوم كذلك بدراسة تقوسات المقاطع المماسة غير المضمحلة للسطوح المعممة شبه المسطرة. نقوم بالإضافة إلى ذلك بالحصول على العلاقات بين التقوسات المقطعية، وتسمى هذه العلاقات: صيغ بلترامي – مونييه مثيل الاقليدية.

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ISSN 1029 - 6069